Uniqueness of a constrained variational problem and large deviations of buffer size

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Abstract

We show global uniqueness of the solution to a class of constrained variational problems, using scaling properties. This is used to establish the essential uniqueness of solutions of a large deviations problem in multiple dimensions. The result is motivated by models of buffers, and in particular the probability of, and typical path to overflow in the limit of small buffers, which we analize.

Index Terms

Uniqueness of Variational problems, Large Deviations, Buffer overflow, AMS model.

I. Introduction

We investigate uniqueness of solutions to variational problems that arise in sample-path large deviations. Our motivation comes from models of buffers in telecommunication systems. The original model was developed by Anick, Mitra, and Sondhi [1]. Weiss [11] cast this model in the framework of sample-path large deviations and showed for the probability of buffer overflow

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}\left(b_n(t) \geq B\right) = \inf_{(\vec{r},T)\in G(B)} \int_0^T \ell(\vec{r}(t), \vec{r}\,'(t))\, dt, \tag{I.1}$$

where $\ell(\vec{r}(t), \vec{r}\,'(t))$ is a positive "cost" function, and $G(B)$ is the set of paths $\vec{r}$ (d-dimensional functions of t) and terminal times T satisfying a buffer overflow property (see § VII). Botvich and Duffield [2] (and independently Courcoubetis and Weber [3] and Simonian and Guibert [7])

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