CCIT Report #647 November 2007

Error Exponents of Erasure/List Decoding Revisited via Moments of Distance Enumerators

Neri Merhav

November 15, 2007

Department of Electrical Engineering
Technion - Israel Institute of Technology
Haifa 32000, ISRAEL

Abstract

The analysis of random coding error exponents pertaining to erasure/list decoding, due to Forney, is revisited. Instead of using Jensen's inequality as well as some other inequalities in the derivation, we demonstrate that an exponentially tight analysis can be carried out by assessing the relevant moments of a certain distance enumerator. The resulting bound has the following advantages: (i) it is at least as tight as Forney's bound, (ii) under certain symmetry conditions associated with the channel and the random coding distribution, it is simpler than Forney's bound in the sense that it involves an optimization over one parameter only (rather than two), and (iii) in certain special cases, like the binary symmetric channel (BSC), the optimum value of this parameter can be found in closed form, and so, there is no need to conduct a numerical search. We have not found yet, however, a numerical example where this new bound is strictly better than Forney's bound. This may provide an additional evidence to support Forney's conjecture that his bound is tight for the average code. We believe that the technique we suggest in this paper can be useful in simplifying, and hopefully also improving, exponential error bounds in other problem settings as well.

Index Terms: random coding, erasure, list, error exponent, distance enumerator.

1 Introduction

In his celebrated paper [3], Forney extended Gallager's bounding techniques [2] and found exponential error bounds for the ensemble performance of optimum generalized decoding rules that include the options of erasure, variable size lists, and decision feedback (see also later studies, e.g., [1], [4],[5], [6], [8], and [10]).