CCIT Report #652 November 2007

Degrees of Freedom Region for the MIMO X Channel

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Abstract

We provide achievability as well as converse results for the degrees of freedom region of a MIMO X channel, i.e., a system with two transmitters, two receivers, each equipped with multiple antennas, where independent messages need to be conveyed over fixed channels from each transmitter to each receiver. The inner and outerbounds on the degrees of freedom region are tight whenever integer degrees of freedom are optimal for each message. With $M = 1$ antennas at each node, we find that the total (sum rate) degrees of freedom are bounded above and below as $1 \leq \eta_X^\star \leq \frac{4}{3}$. If $M > 1$ and channel matrices are non-degenerate then the precise degrees of freedom $\eta_X^\star = \frac{4}{3}M$. Thus, the MIMO X channel has non-integer degrees of freedom when M is not a multiple of 3. Simple zero forcing without dirty paper encoding or successive decoding, suffices to achieve the $\frac{4}{3}M$ degrees of freedom. The key idea for the achievability of the degrees of freedom is *interference alignment* - i.e., signal spaces are aligned at receivers where they constitute interference while they are separable at receivers where they are desired. With equal number of antennas at all nodes, we explore the increase in degrees of freedom when some of the messages are made available to a transmitter or receiver in the manner of cognitive radio. With a cognitive transmitter, i.e. with one message shared between transmitters on the MIMO X channel we show that the number of degrees of freedom $\eta = \frac{3}{2}M$ (for $M > 1$). The same degrees of freedom are obtained on the MIMO X channel with a cognitive receiver as well, i.e. when one message is made available to its non-intended receiver. In contrast to the X channel result, we show that for the MIMO *interference* channel, the degrees of freedom are not increased even if both the transmitter and the receiver of one user know the other user's message. However, the interference channel can achieve the full $2M$ degrees of freedom if *each* user has either a cognitive transmitter or a cognitive receiver. Lastly, if the channels vary with time/frequency then the X channel with single antennas $(M = 1)$ at all nodes has exactly $4/3$ degrees of freedom with no shared messages and exactly $3/2$ degrees of freedom with a cognitive transmitter or a cognitive receiver.