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# On the Compound MIMO Broadcast Channel

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***Abstract*— We consider the Gaussian multi-antenna compound broadcast channel where one transmitter transmits several messages, each intended for a different user whose channel realization is arbitrarily chosen from a finite set. Our investigation focuses on the behavior of this channel at high SNRs and we obtain the multiplexing gain of the sum capacity for a number of cases, and point out some implications of the total achievable multiplexing gain region.**[1]

## I. Introduction

With the advent of 3rd generation cellular systems, multi-antenna systems are becoming common place. Even though many theoretical questions on the downlink channel (alternatively, broadcast channel, BC) in general are still open, the capacity region of multi-antenna downlink channel as well as some other questions have been resolved [1]. However, many practical questions still remain open. For example, the capacity region of a fading BC (scalar as well as multi-antenna) with no or partial channel state information (CSI) at the transmitter [2]. Another open problem is that of the capacity region of a multi-antenna BC with private and common messages. Recently, these questions have attracted attention [3], [4], [5], [6], [7]. In this paper we address and give theoretical bounds for a related and yet unsolved problem of a compound multi-antenna BC.

We consider a memoryless compound multi-antenna BC and focus on the case where the transmitter has $M$ transmit antennas and each of the receivers has only one receive antenna. More precisely, we assume that the fading vector of user $i$ takes one of $J_i$ (finite) values. In addition, the transmitter has precise knowledge of all $J_i$ fading vectors but not of the index of the actual realization of the fading vector. Therefore, a time sample of the channel can be defined as follows:

$$y_i^j = \mathbf{h}_i^{j\dagger}\mathbf{x} + n_i^j \quad i=1,\ldots,K \quad j=1,\ldots,J_i \qquad (1)$$

where

- $\mathbf{x}$ is a complex input vector. We assume that the input is power limited such that $E\mathbf{x}^\dagger\mathbf{x} \le P$.
- $y_i^j$ is the signal received by the $j$'th realization/instance of user $i$.
- $n_i^j \sim \mathcal{CN}(0,1)$ is an additive white circularly symmetric Gaussian noise which is present at the $j$'th realization/instance of user $i$.
- $K$ is the number of users and hence, the number of different messages to be simultaneously transmitted.
- $\mathbf{h}_i^j$ is the complex fading vector of the $j$'th realization/instance of the $i$'th user.

Each of the receivers has exact knowledge of the actual realization of the channel. We wish to transmit in such a manner that no matter what is the actual realization of the users' channels ($\mathbf{h}_i^1$ or $\mathbf{h}_i^2$ or ... or $\mathbf{h}_i^{J_i}$), our transmitter will be able to successfully send its messages.

The case where the channel is scalar, i.e. $M=1$, is well understood. As in the case of $M=1$ the channel is degraded, the capacity is determined by the worst realization (with the smallest fading realization $|h_i^j|$) of each user group. Thus, the capacity region is that of a scalar Gaussian BC with one realization per message as follows:

$$R_{\pi_i} \le \log\left(1+\frac{\gamma_{\pi_i}P_{\pi_i}}{1+\gamma_{\pi_i}\sum_{l=1}^{i-1}P_{\pi_l}}\right), \qquad i=1,\ldots,K \quad (2)$$

where $P_i$'s are the power allocations per user such that $\sum_{i=1}^K P_i = P$. The $\gamma_i = \min_{j=1\ldots,J_i}|\mathbf{h}_i^j|^2$ are the fading power of the worst realization of user $i$ and $\pi_i$ is a permutation matrix which orders the users according to $\gamma_i$ from the largest to the smallest one.

However, we consider the case where $M>1$ which is not degraded and the capacity region of this problem is yet unknown. Therefore, we concentrate on the high SNR regime and obtain new results regarding the multiplexing gain of the sum-capacity of the above compound BC for a number of cases. We use the sum-capacity as a measure of the capabilities of the channel and define the multiplexing gain as the maximum value of

$$\lim_{P\to\infty}\frac{R_1+R_2+\ldots R_K}{\log(P)}$$

where the maximum is taken over all transmission strategies.

An alternative view of this channel is that of a broadcast channel with common messages. The different realizations of the channel can be considered as different users to which a common message is being transmitted. This is actually quite a realistic model as third generation cellular systems transmit TV broadcasts over the downlink channel [8]. This application also motivates the consideration of the high SNR regime, where the impact of multi-antenna downlink systems is more pronounced.

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