CCIT Report #676
December 2007

# The Generalized Random Energy Model of Spin Glasses and its Application to the Statistical Physics of Code Ensembles with Hierarchical Structures

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**Abstract**

In an earlier work, the statistical physics associated with finite–temperature decoding of code ensembles, along with the relation to their random coding error exponents, were explored in a framework that is analogous to Derrida's random energy model (REM) of spin glasses, according to which the energy levels of the various spin configurations are independent random variables. The generalized REM (GREM) extends the REM in that it introduces correlations between energy levels in an hierarchical structure. In this paper, we explore some analogies between the behavior of the GREM and that of code ensembles which have parallel hierarchical structures. In particular, in analogy to the fact that the GREM may have different types of phase transition effects, depending on the parameters of the model, then the above–mentioned hierarchical code ensembles behave substantially differently in the various domains of the design parameters of these codes. We make an attempt to explore the insights that can be imported from the statistical mechanics of the GREM and be harnessed to serve for code design considerations and guidelines.

**Index Terms:** Spin glasses, GREM, phase transitions, random coding, error exponents.

## 1 Introduction

In the last few decades it has become apparent that many problems in Information Theory have analogies to certain problems in the area of statistical physics of disordered systems. Such analogies are useful because physical insights, as well as statistical mechanical tools and analysis techniques can be harnessed in order to advance the knowledge and the understanding with regard to the information–theoretic problem under discussion.