

Reduce and Boost: Recovering Arbitrary Sets of Jointly Sparse Vectors

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Abstract

The rapid developing area of compressed sensing suggests that a sparse vector lying in an arbitrary high dimensional space can be accurately recovered from only a small set of non-adaptive linear measurements. Under appropriate conditions on the measurement matrix, the entire information about the original sparse vector is captured in the measurements, and can be recovered using efficient polynomial methods. The vector model has been extended both theoretically and practically to a finite set of sparse vectors sharing a common non-zero location set. In this paper, we treat a broader framework in which the goal is to recover a possibly infinite set of jointly sparse vectors. Extending existing recovery methods to this model is difficult due to the infinite structure of the sparse vector set. Instead, we prove that the entire infinite set of sparse vectors can be recovered by solving a single, reduced-size finite-dimensional problem, corresponding to recovery of a finite set of sparse vectors. We then show that the problem can be further reduced to the basic recovery of a single sparse vector by randomly combining the measurement vectors. Our approach results in exact recovery of both countable and uncountable sets as it does not rely on discretization or heuristic techniques. To efficiently recover the single sparse vector produced by the last reduction step, we suggest an empirical boosting strategy that improves the recovery ability of any given sub-optimal method for recovering a sparse vector. Numerical experiments on random data demonstrate that when applied to infinite sets our strategy outperforms discretization techniques in terms of both run time and empirical recovery rate. In the finite model, our boosting algorithm is characterized by fast run time and superior recovery rate than known popular methods.

Index Terms

Basis pursuit, compressed sensing, multiple measurement vectors (MMV), orthogonal matching pursuit (OMP), sparse representation.

I. INTRODUCTION

Many signals of interest often have sparse representations, meaning that the signal is well approximated by only a few large coefficients in a specific basis. The traditional strategy to capitalize on the sparsity profile is to