CCIT Report #691 March 2008

The Random Energy Model in a Magnetic Field and Joint Source–Channel Coding

Neri Merhav

Department of Electrical Engineering
Technion - Israel Institute of Technology
Haifa 32000, ISRAEL

Abstract

We demonstrate that there is an intimate relationship between the magnetic properties of Derrida's random energy model (REM) of spin glasses and the problem of joint source–channel coding in Information Theory. In particular, typical patterns of erroneously decoded messages in the coding problem have "magnetization" properties that are analogous to those of the REM in certain phases, where the non–uniformity of the distribution of the source in the coding problem, plays the role of an external magnetic field applied to the REM. We also relate the ensemble performance (random coding exponents) of joint source–channel codes to the free energy of the REM in its different phases.

Keywords: spin glasses, REM, phase transitions, magnetization, information theory, joint source–channel codes.

1 Introduction

In the last few decades it has become apparent that many problems in Information Theory, and coding problems in particular, can be mapped onto (and interpreted as) analogous problems in the area of statistical physics of disordered systems, most notably, spin glass models. Such analogies are useful because physical insights, as well as statistical mechanical tools and analysis techniques (like the replica method), can be harnessed in order to advance the knowledge and the understanding with regard to the information–theoretic problem under discussion (and conversely, information–theoretic approaches to problems in physics may sometimes prove useful to physcists as well). A very small, and by no means exhaustive, sample of works along this line includes references [1]–[25].