CCIT Report #694 April 2008

Symmetric Hill Order – an Optimal Solution of a Linear Ordering Adjacency Problem

Shmuel Wimer [a,b] , Konstantin Moiseev [a] and Avinoam Kolodny [a]

[a] Electrical Engineering Department, Technion, Haifa, Israel

[b] Intel Corporation, Israel Development Center, Haifa Israel

Abstract

This paper solves a type of Linear Ordering Problem (LOP) which arises in VLSI interconnect design. Let $f(x,y):\mathbb{R}^2 \to \mathbb{R}$ be symmetric and $f \in \mathrm{D}^2$, satisfying $\partial f(x,y)/\partial x \times \partial f(x,y)/\partial x > 0$ and $\partial^2 f(x,y)/\partial x \partial y < 0$. The LOP solved comprises n objects, each associated with a real value parameter r_i, $1 \le i \le n$, and a cost associated any two objects defined by $f(r_i, r_j)$ if $|\pi(i) - \pi(j)| = 1$, $1 \le i, j \le n$, and zero otherwise. We show that the permutation π which minimizes the total cost $\sum_{i=1}^{n-1} f\left(r_{\pi(i)}, r_{\pi(i)+1}\right)$ is determined upfront by the relations between the parameter values r_i. Such permutation is called "***symmetric hill***". It generalizes a family of well known problems arising in interconnect design of VLSI circuits where objects are parallel wires and the cost reflects various design metrics such as delay, power consumption or cross-coupling noise.

1. Introduction and motivation

Let $f(x,y):\mathbb{R}^2 \to \mathbb{R}$ be symmetric and $f \in \mathrm{D}^2$, satisfying $\partial f(x,y)/\partial x \times \partial f(x,y)/\partial x > 0$ and $\partial^2 f(x,y)/\partial x \partial y < 0$. In the rest of the paper $f(x,y)$ will assume these properties without further mention. Let $(r_1, \ldots, r_n)$ be a sequence of n real non negative numbers associated with n objects, let Π be the set of all permutations $\pi : \{1,\ldots,n\} \to \{1,\ldots,n\}$, and we denote by $\left(r_{\pi(1)}, \ldots, r_{\pi(n)}\right)$ the sequence obtained by applying π to the original set. This paper explores the problem of finding $\pi^* \in \Pi$ which minimizes the sum of costs defined for any two adjacent objects: