Gallager-Type Bounds for Non-Binary Linear Block Codes over Memoryless Symmetric Channels

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Abstract

The performance of non-binary linear block codes is studied in this paper via the derivation of new upper bounds on the error probability under ML decoding. The transmission of these codes is assumed to take place over a memoryless and symmetric channel. The new bounds, which rely on the Gallager bounding technique, are applied to expurgated ensembles of non-binary and regular low-density parity-check (LDPC) codes. These upper bounds are also compared with classical and recent sphere-packing lower bounds. This study indicates that the new upper bounds are useful for the performance evaluation of coded communication systems which incorporate non-binary coding techniques.

Index Terms

Block codes, linear codes, low-density parity-check (LDPC) codes, ML decoding, non-binary codes, sphere-packing bounds.

I. Introduction

The performance of coded communication systems is usually analyzed via upper and lower bounds on the decoding error probability. These bounds are of interest since the performance analysis of coded communication systems rarely admits exact expressions. Modern coding schemes (e.g., codes defined on graphs) perform reliably at rates which are close to the channel capacity, whereas union bounds are useless for codes of moderate to large block lengths at rates above the channel cut-off rate. The limitation of the union bound therefore motivates the introduction of some improved bounding techniques which can be also efficiently calculated. Although the performance analysis of specific codes is in general prohibitively complex, this kind of analysis is tractable for various code ensembles for which the derivation of some of their basic features (e.g., the average distance spectrum) lends itself to analysis. For a comprehensive tutorial on the performance analysis of binary linear block codes under maximum-likelihood (ML) decoding, the reader is referred to [1] and references therein, whereas this work is focused on non-binary linear block codes.

The 1965 Gallager bound [2] is one of the well-known upper bounds on the decoding error probability of ensembles of fully random block codes, and it is informative at all rates below the channel capacity limit. Emerging from the 1965 Gallager bound, the bounds of Duman and Salehi (see [3] and [4]) possess the pleasing feature that they are amenable to analysis for specific codes and general code ensembles for which the (average) distance spectrum analysis is tractable.

The framework of the Duman and Salehi bounding technique, in particular its second version (called hereafter the 'DS2 bound'), is generalized in [1], [5] and [6] for various memoryless communication systems. Moreover, this bound facilitates the derivation of a large class of previously reported bounds (or their Chernoff versions), as

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