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Beyond Bandlimited Sampling: Nonlinearities, Smoothness and Sparsity

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Digital applications have developed rapidly over the last few decades. Since many sources of information are of analog or continuous-time nature, discrete-time signal processing (DSP) inherently relies on sampling a continuous-time signal to obtain a discrete-time representation. Consequently, sampling theories lie at the heart of signal processing applications and communication systems. A few examples are sampling rate conversion between audio formats and for software radio [1], biomedical imaging [2], lens distortion correction, super-resolution of image sequences and more.

To accommodate high operating rates while retaining low computational cost, efficient analog-to-digital (ADC) and digital-to-analog (DAC) converters must be developed. Many of the limitations encountered in current converters is due to a traditional assumption that the sampling stage must acquire the data at the Shannon-Nyquist rate, corresponding to twice the signal bandwidth [3], [4], [5]. To avoid aliasing, a sharp low-pass filter must be implemented prior to sampling. The reconstructed signal is also a bandlimited function, generated by integer shifts of the sinc interpolation kernel.

A major drawback of this paradigm is that many natural signals are better represented in alternative bases other than the Fourier basis [6], [7], [8], or possess further structure in the Fourier domain. In addition, ideal point-wise sampling, as assumed by the Shannon theorem, cannot be implemented. More practical ADCs introduce a distortion which should be accounted for in the reconstruction process. Finally, implementing the infinite sinc interpolating kernel is difficult, since it has slow decay. In practice, much simpler kernels are used such as linear interpolation. Therefore there is a need to develop a general sampling theory that will accommodate an extended class of signals beyond bandlimited functions, and will account for the nonideal nature of the sampling and reconstruction processes.

Sampling theory has benefited from a surge of research in recent years, due in part to the intense research in wavelet theory and the connections made between the two fields. In this survey we present several extensions of the Shannon theorem, that have been developed primarily in the past two decades, which treat a wide class of input signals as well as nonideal sampling and nonlinear distortions. This framework is based on viewing sampling in a broader sense of projection onto appropriate subspaces, and then choosing the subspaces to yield interesting

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