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GEOMETRIC APPROACH TO SAMPLING AND COMMUNICATION

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Abstract. Relationships that exists between the classical, Shannon-type, and geometric-based approach to sampling are investigated. Some aspects of coding and communication through a Gaussian channel problem are considered. In particular, a constructive method to determine the quantizing dimension in Zador's theorem is provided. A geometric version of Shannon's Second Theorem is introduced. Applications to Pulse Code Modulation and Vector Quantization of Images are provided. In addition, we sketch the geometerization of wavelets for Image Processing purposes. We also discuss the implications of the Uncertainty Principle on sampling and reconstruction of images. An extension of our sampling scheme to a certain class of infinite dimensional manifolds is considered.

1. General background

1.1. **Introduction.** We consider a geometric approach to Shannon's sampling theorem, i.e. one based on sampling the *graph* of the signal, considered as a *manifold*, rather than a sampling of the *domain* of the signal, as is customary in both theoretical and applied signal and image processing, motivated by the framework of harmonic analysis. In this context it is important to note that Shannon's original intuition was deeply rooted in the geometric approach, as exposed in his seminal work [60], and also in [61]. Indeed, it is this geometric viewpoint of the problem which distinguishes Shannon from Kotelnikov [35] and Nyquist [45], and allows him to transcend the restricted context of technical communication theory. We were also inspired in our endeavor by the "dictionary" of geometric to communication theory notions, and we strived to emulate it.[1]

Our approach is based upon the following sampling theorem for differentiable manifolds that was recently presented and applied in the context image processing ([54], [55])[2]:

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[1]Different paths towards the geometerization of Sampling Theory can be found, e.g. in [49] and [34].

[2]An approach similar to ours appeared in [37], however mathematically less rigorous and comprehensive. Unfortunately, we were not aware of the existence of this study upon