CCIT REPORT #709 November 2008

Joint Source Channel Coding for Fading Channel: The Broadcast Approach-Expected Distortion, Gaussian Source and Channel

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***Abstract*— We consider the problem of transmitting a Gaussian source on a slowly fading Gaussian channel, subject to the mean squared error distortion measure. The channel state information is known only at the receiver but not the transmitter. The source is assumed to be encoded in a successive refinement manner, and then transmitted over the channel using the broadcast strategy. In order to minimize the expected distortion at the receiver, optimal power allocation is essential. We propose an efficient algorithm to compute the optimal solution in linear time $O(M)$. Moreover, we provide a derivation of the optimal power allocation when the fading state is a continuum, using the classical variational method. The proposed algorithm as well as the continuous solution is based on an alternative representation of the capacity region of the Gaussian broadcast channel.**

I. Introduction

Fading channel occurs naturally as a model in wireless communications. For slow fading, the receiver can usually recover the channel state information (CSI) accurately, however the transmitter only knows the probability distribution of CSI, but not the realization. Such uncertainty can cause significant performance degradation, and the broadcast strategy was used in [1] as an approach to combat this effect. In this strategy, some information can only be decoded when the fading is less severe, which is superimposed on the information that can be decoded under more severe fading. Thus the receiver can decode the information adaptively, according to the realization of the channel state. The similarity to the degraded broadcast channel [2] is clear in this context, particularly for channels with finite number of fading states. Generalizing this view, when the fading gain can take continuous values, the receiver can be taken as a continuum of users in a broadcast channel.

The broadcast strategy naturally matches the successive refinement (SR) source coding framework [3], as the information decodable under the strongest fading is protected the most, and should be used to convey the base layer information in the SR coding. As more information can be decoded when the channel is subject to less fading, more SR coded layers can be decoded, and the reconstruction quality improves. In this work, we consider this scenario for a quadratic Gaussian source. In order to minimize the expected distortion at the receiver, it is essential to find the optimal power allocation in the broadcast strategy, and this is indeed our focus. This cross layer design approach was in fact already suggested in [1].

Initial effort on this problem was made by Sesia *et al.* in [4], where the broadcast strategy coupled with SR source coding was compared with several other schemes. Etemadi and Jafarkhani also considered this problem in [5], and provided an iterative algorithm, by separating the optimization problem into two sub-problems. In two more recent works [6] [7], Ng *et al.* provided a recursive algorithm to compute the optimal power allocation for M fading states, with worst case complexity of $O(2^M)$; moreover, by directly taking the limit of the optimal solution for the discrete case, a solution was given for the continuous case optimal power allocation, under the assumption that the optimal power allocation is concentrated in a single interval. Similar problems were considered in [8] [9] in the high SNR regime from the perspective of distortion exponent.

Our contribution in the present work is two-fold: firstly, we propose a new algorithm that computes the optimal power allocation for M fading states with $O(M)$ complexity, i.e., in linear time; secondly, we provide a derivation of the continuous case optimal power allocation solution by the classical variational method [10]. Both the algorithm and the derivation rely on an alternative representation of the Gaussian broadcast channel capacity, which appeared in [11].

The rest of the paper is organized as follows. In Section II we give the system model, and in Section III the new algorithm is provided and its optimality is proved. In Section IV we give the derivation for the continuous case solution, and Section V concludes the paper.

II. System model and Preliminaries

We assume the memoryless source $\{X_i\}_{i=1}^{\infty}$ is generated independently and identically according to a zero-mean unit variance Gaussian distribution. To simplify the notation, we directly assume the channel in the real domain as

$$Y_c = \sqrt{s} X_c + N, \tag{1}$$

where X_c is the real-valued channel input and Y_c is the channel output, $s \in \mathbb{R}$ is the (random) channel power gain, and N is the zero-mean unit variance Gaussian additive noise in the channel. Extension to complex system with circular symmetric complex noise is straightforward.

We consider a slowly fading channel model, where each channel codeword consists of a length-l_c channel symbol block. Source symbols of block length-l_s is encoded into a single channel codeword, and there is a source channel mismatch factor $b = l_c/l_s$; see also Fig. 1. Each channel block is assumed to be sufficiently long to approach channel