

On Successive Refinement for the Kaspi/Heegard-Berger Problem

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Abstract

Consider a source that produces independent copies of a triplet of jointly distributed random variables, $\{X_i, Y_i, Z_i\}_{i=1}^{\infty}$. The process $\{X_i\}$ is observed at the encoder, and is supposed to be reproduced at two decoders, decoder Y and decoder Z, where $\{Y_i\}$ and $\{Z_i\}$ are observed, respectively, in either a causal or non-causal manner. The communication between the encoder and the decoders is carried in two successive stages. In the first stage, the transmission is available to both decoders and they reconstruct the source according to the received bit-stream and the individual side information ($\{Z_i\}$ or $\{Y_i\}$). In the second stage, additional information is sent to both decoders and they refine the reconstructions of the source according to the available side information and the transmissions at both stages. It is desired to find the necessary and sufficient conditions on the communication rates between the encoder and decoders, so that the distortions incurred (at each stage) will not exceed given thresholds. For the case of non-degraded causal side information at the decoders, an exact single-letter characterization of the achievable region for is derived for the case of pure source-coding. Then, for the case of communication between the encoder and decoders carried over independent memoryless discrete channels with random states known causally/non-causally at the encoder and with causal side information about the source at the decoders, a single-letter characterization of all achievable distortion in both stages is provided and it is shown that the separation theorem holds. Finally, for non-causal degraded side information, inner and outer bounds to the achievable rate-distortion region are derived. These bounds are shown to be tight for certain cases of reconstruction requirements at the decoders, thereby shading some light on the problem of successive refinement with non-degraded side information at the decoders.

Index terms - causal/non-causal side information, channel capacity, degraded side-information, joint source-channel coding, separation theorem, source coding, successive refinement.

1 Introduction

We consider an instance of the multiple description problem, which is successive refinement (SR) of information. The term “successive refinement of information” is applicable