

# Statistical Physics of Signal Estimation in Gaussian Noise: Theory and Examples of Phase Transitions

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## Abstract

We consider the problem of signal estimation (denoising) from a statistical mechanical perspective, using a relationship between the minimum mean square error (MMSE), of estimating a signal, and the mutual information between this signal and its noisy version. The paper consists of essentially two parts. In the first, we derive several statistical–mechanical relationships between a few important quantities in this problem area, such as the MMSE, the differential entropy, the Fisher information, the free energy, and a generalized notion of temperature. We also draw analogies and differences between certain relations pertaining to the estimation problem and the parallel relations in thermodynamics and statistical physics. In the second part of the paper, we provide several application examples, where we demonstrate how certain analysis tools that are customary in statistical physics, prove useful in the analysis of the MMSE. In most of these examples, the corresponding statistical–mechanical systems turn out to consist of strong interactions that cause phase transitions, which in turn are reflected as irregularities and discontinuities (similar to threshold effects) in the behavior of the MMSE.

## Index Terms

Gaussian channel, denoising, de Bruijn’s identity, MMSE estimation, phase transitions, random energy model, spin glasses, statistical mechanics.

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