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Xampling—Part I: Practice

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***Abstract*—We introduce Xampling, a design methodology for sub-Nyquist sampling of continuous-time analog signals. The main principles underlying this framework are the ability to capture a broad signal model, low sampling rate, efficient analog and digital implementation and lowrate baseband processing. The main hypothesis of Xampling is that in order to break through the Nyquist barrier, one has to combine classic methods and results from sampling theory together with recent developments from the literature of compressed sensing. In this paper, we present the Xampling framework and examine several sub-Nyquist approaches in light of the four Xampling principles. It is shown that previous methods suffer from analog implementation issues, large computational loads in the digital domain, and have no baseband processing capabilities. An exception is the recently proposed modulated wideband converter (MWC) which satisfies the model, rate and implementation criteria, though lacking the baseband processing capability. Here, we extend the MWC by proposing a digital algorithm which extracts each band of the signal from the compressed measurements, thus enabling lowrate (baseband) processing. The converter with the proposed algorithm conforms with the Xampling desiderata. In addition, we describe two configurations of the converter for efficient spectrum sensing in wideband cognitive radio receivers. In the second part of this work we study theoretical aspects of rate and stability of sub-Nyquist systems, following the pragmatic theme of the Xampling methodology.**

***Index Terms*—Baseband processing, cognitive radio, compressed sensing, modulated wideband converter, sub-Nyquist sampling, Xampling.**

I. Introduction

SIGNAL processing methods have changed substantially over the last several decades. The number of operations that are shifted from analog to digital is constantly increasing, leaving amplifications and fine tunings to the traditional front-end. In the chain of sampling, processing and reconstruction, the conversion to digital has become a serious bottleneck. While technology advances enable mass processing of huge data streams, the acquisition capabilities do not scale sufficiently fast [1]. For some applications, the maximal frequency of the input signals, which dictates the Nyquist rate, already exceeds the possible rates achievable with existing devices. Sampling theory, the gate to the digital world, is needed to break through the rate bottleneck.

Consider the scenario depicted in Fig. 1, which is prevalent in communication systems. A few narrowband transmissions are modulated onto carrier frequencies f_i, which can take on any value below $f_{\max}$. This leads to a *multiband* spectral support that occupies only a small portion of the wideband spectrum defined by $f_{\max}$. The receiver converts each transmission to digital by demodulating the carrier frequencies f_i. Once the transmission contents appear at baseband, that is near the origin, they are lowpass filtered and sampled at a low rate. In the example, the three concurrent transmissions result in a signal $x(t)$ which is supported on $N = 6$ frequency intervals, or bands, each of width not greater than B Hz. This approach leads to sampling at a rate that is proportional to NB, rather than to the radio-frequency (RF) $f_{\max}$, which can be prohibitively large in modern applications. Depending on the modulation technique, the information, either a bit steam or an analog message, is extracted from the samples. Often, this operation involves a matched filter.

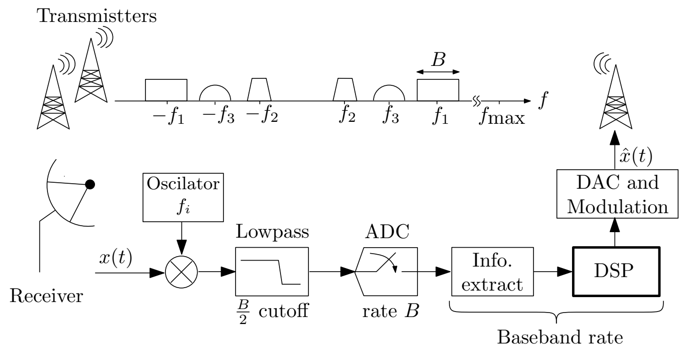

Fig. 1. Three RF transmissions with different carriers f_i. The receiver demodulates each transmission separately and samples the baseband version.

Digital signal processing (DSP) is the crowning glory of the chain of blocks in Fig. 1. The prime goal of analog to digital conversion (ADC) is isolating the delicate interaction with the continuous world, so that sophisticated algorithms can be developed in a flexible software environment. Digital filtering, channel equalization, system identification, blind source separation, noise shaping and a rich variety of software algorithms – all lie under the DSP block of Fig. 1. Besides processing, reconstruction of the input $x(t)$ can be obtained by digital to analog conversion (DAC) and remodulating onto the original carriers f_i. This option is useful in relay stations that re-transmit the input after local improvements to the signal. All digital computations are carried out at the actual information rate, which is referred to hereafter as *baseband processing*.

Utilizing the scheme of Fig. 1 requires knowing the carrier frequencies f_i. As explained in Section II, this approach can tolerate only slight deviations from the prespecified carrier values f_i and cannot extend to arbitrary spectral support. Classic works in sampling theory [2]–[6] study periodic nonuniform sampling as an alternative, though these solutions also rely on knowledge of the carrier frequencies f_i. The literature describes several sub-Nyquist strategies, other than Fig. 1, that have the potential to treat arbitrary carrier positions: multi-

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