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Optimum Estimation via Gradients of Partition Functions and Information Measures: A Statistical–Mechanical Perspective

Neri Merhav

Department of Electrical Engineering
Technion – Israel Institute of Technology
Technion City, Haifa 32000, Israel
merhav@ee.technion.ac.il

Abstract

In continuation to a recent work on the statistical–mechanical analysis of minimum mean square error (MMSE) estimation in Gaussian noise via its relation to the mutual information (the I–MMSE relation), here we propose a simple and more direct relationship between optimum estimation and certain information measures (e.g., the information density and the Fisher information), which can be viewed as partition functions and hence are amenable to analysis using statistical–mechanical techniques. The proposed approach has several advantages, most notably, its applicability to general sources and channels, as opposed to the I–MMSE relation and its variants which hold only for certain classes of channels (e.g., additive white Gaussian noise channels). We then demonstrate the derivation of the conditional mean estimator and the MMSE in a few examples. Two of these examples turn out to be generalizable to a fairly wide class of sources and channels. For this class, the proposed approach is shown to yield an approximate conditional mean estimator and an MMSE formula that has the flavor of a single–letter expression. We also show how our approach can easily be generalized to situations of mismatched estimation.

Index Terms: Conditional mean estimation, minimum mean squared error, partition function, statistical mechanics, Fisher information.

1 Introduction

Relationships between signal estimation, signal detection, and information measures, both in discrete time and continuous time, have been known for decades [1],[3],[8] and have gained a remarkable degree of revived interest and research activity in the last several years, see, e.g., [4], [5], [6], [7], [11], [12], [13] and references therein.