

Bose–Einstein Condensation in the Large Deviations Regime with Applications to Information System Models

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Abstract. We study the large deviations behavior of systems that admit a certain form of a product distribution, which is frequently encountered both in Physics and in various information system models. First, to fix ideas, we demonstrate a simple calculation of the large deviations rate function for a single constraint (event). Under certain conditions, the behavior of this function is shown to exhibit an analogue of Bose–Einstein condensation (BEC). More interestingly, we also study the large deviations rate function associated with two constraints (and the extension to any number of constraints is conceptually straightforward). The phase diagram of this rate function is shown to exhibit as many as seven phases, and it suggests a two–dimensional generalization of the notion of BEC (or more generally, a multi–dimensional BEC). While the results are illustrated for a simple model, the underlying principles are actually rather general. We also discuss several applications and implications pertaining to information system models.