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Data Processing Theorems and the Second Law of Thermodynamics

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Abstract

We draw relationships between the generalized data processing theorems of Ziv and Zakai (1973 and 1975) and the dynamical version of the second law of thermodynamics, a.k.a. the Boltzmann H–Theorem, which asserts that the Shannon entropy, $H(X_t)$, pertaining to a finite–state Markov process $\{X_t\}$, is monotonically non–decreasing as a function of time t, provided that the steady–state distribution of this process is uniform across the state space (which is the case when the process designates an isolated system). It turns out that both the Ziv–Zakai generalized data processing theorems and the Boltzmann H–Theorem can be viewed as special cases of a more general principle concerning the monotonicity (in time) of a certain generalized information measure applied to a Markov process. This gives rise to a new look at the Ziv–Zakai data processing theorem, which suggests to exploit certain degrees of freedom that may lead to better bounds, for a given choice of the convex function that defines the generalized mutual information.