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On Optimum Strategies for Minimizing the Exponential Moments of a Given Cost Function

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Abstract

We consider a general problem of finding a strategy that minimizes the exponential moment of a given cost function, with an emphasis on its relation to the more common criterion of minimization the expectation of the first moment of the same cost function. In particular, our main result is a theorem that gives simple sufficient conditions for a strategy to be optimum in the exponential moment sense. This theorem may be useful in various situations, and application examples are given. We also examine the asymptotic regime and investigate universal asymptotically optimum strategies in light of the aforementioned sufficient conditions, as well as phenomena of irregularities, or phase transitions, in the behavior of the asymptotic performance, which can be viewed and understood from a statistical–mechanical perspective. Finally, we propose a new route for deriving lower bounds on exponential moments of certain cost functions (like the square error in estimation problems) on the basis of well known lower bounds on their expectations.

Index Terms: loss function, exponential moment, large deviations, phase transitions, universal schemes.

1 Introduction

Many problems in information theory, communications, statistical signal processing, and related disciplines can be formalized as being about the quest for a strategy s that minimizes (or maximizes) the expectation of a certain cost function, $\ell(X,s)$, where X is a random variable (or a random vector). Just a few examples of this generic paradigm are the following: (i) Lossless and lossy data compression, where X symbolizes the data to be compressed, s is the data compression scheme, and $\ell(X,s)$ is the length of the compressed binary representation, or the distortion (in the lossy