CCIT Report #794 August 2011

Structure Theorems for Real–Time Variable Rate Coding With and Without Side Information

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Abstract

The output of a discrete Markov source is to be encoded instantaneously by a variable–rate encoder and decoded by a finite–state decoder. Our performance measure is a linear combination of the distortion and the instantaneous rate. Structure theorems, pertaining to the encoder and next–state functions are derived for every given finite–state decoder, which can have access to side information.

I. Introduction

We consider the following source coding problem. Symbols produced by a discrete Markov source are to be encoded, transmitted noiselessly and reproduced by a decoder which can have causal access to side information (SI) correlated to the source. Operation is in real time, that is, the encoding of each symbol and its reproduction by the decoder must be performed without any delay and the distortion measure does not tolerate delays.

The decoder is assumed to be a finite–state machine with a fixed number of states. With no SI, the scenario where the encoder is of fixed rate was investigated by Witsenhausen [1]. It was shown that for a given decoder, in order to minimize the distortion at each stage for a Markov source of order k, an optimal encoder can be found among those for which the encoding function depends on the k last source symbols and the decoder's state (in contrast to the general case where its a function of all past source symbols). Walrand and Varaiya [2] extended this finding to a joint source–channel setup with noiseless feedback. Teneketzis [3] used ideas from both [1] and [2] and considered the joint source–channel setup for a given finite state decoder without feedback. A causal variant of the Wyner Ziv problem [4] was also considered by Teneketzis [3]. It is shown in [3] that the optimal (fixed rate) encoder for this case is a function of the current source symbol and the probability mass function of the decoder's state for the symbols sent so far. Borkar, Mitter and Tatikonda [5] derived structure theorems of a similar spirit when the cost function is a linear combination (Lagrangian) of the conditional entropy of the reproduction sequence and the mean square error of the reproduction. The scenario where the encoder is also a finite state machine was considered by Gaarder and Slepian in [6]. In some cases, the minimization of the distortion (or cost) can be cast as a stochastic control problem. In this case, tools developed for Markov decision processes