CCIT Report #806 March 2012

On Optimum Parameter Modulation–Estimation From a Large Deviations Perspective

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Abstract

We consider the problem of jointly optimum modulation and estimation of a real–valued random parameter, conveyed over an additive white Gaussian noise (AWGN) channel, where the performance metric is the large deviations behavior of the estimator, namely, the exponential decay rate (as a function of the observation time) of the probability that the estimation error would exceed a certain threshold. Our basic result is in providing an exact characterization of the fastest achievable exponential decay rate, among all possible modulator–estimator (transmitter–receiver) pairs, where the modulator is limited only in the signal power, but not in bandwidth. This exponential rate turns out to be given by the reliability function of the AWGN channel. We also discuss several ways to achieve this optimum performance, and one of them is based on quantization of the parameter, followed by optimum channel coding and modulation, which gives rise to a separation–based transmitter, if one views this setting from the perspective of joint source–channel coding. This is in spite of the fact that, in general, when error exponents are considered, the source–channel separation theorem does not hold true. We also discuss several observations, modifications and extensions of this result in several directions, including other channels, and the case of multidimensional parameter vectors. One of our findings concerning the latter, is that there is an abrupt threshold effect in the dimensionality of the parameter vector: below a certain critical dimension, the probability of excess estimation error may still decay exponentially, but beyond this value, it must converge to unity.

Index Terms: Parameter estimation, modulation, AWGN, threshold effect, large deviations, reliability function, error exponents.