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Exponential Error Bounds on Parameter Modulation–Estimation for Discrete Memoryless Channels

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Abstract

We consider the problem of modulation and estimation of a random parameter U to be conveyed across a discrete memoryless channel. Upper and lower bounds are derived for the best achievable exponential decay rate of a general moment of the estimation error, $\boldsymbol{E}|\hat{U}-U|^{\rho}$, $\rho \geq 0$, when both the modulator and the estimator are subjected to optimization. These exponential error bounds turn out to be intimately related to error exponents of channel coding and to channel capacity. While in general, there is some gap between the upper and the lower bound, they asymptotically coincide both for very small and for very large values of the moment power ρ. This means that our achievability scheme, which is based on simple quantization of U followed by channel coding, is nearly optimum in both limits. Some additional properties of the bounds are discussed and demonstrated, and finally, an extension to the case of a multidimensional parameter vector is outlined, with the principal conclusion that our upper and lower bound asymptotically coincide also for a high dimensionality.

Index Terms: Parameter estimation, modulation, discrete memoryless channels, error exponents, random coding, data processing theorem.