

Another Look at Expurgated Bounds and Their Statistical–Mechanical Interpretation*

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Abstract

We revisit the derivation of expurgated error exponents using a method of type class enumeration, which is inspired by statistical–mechanical methods, and which has already been used in the derivation of random coding exponents in several other scenarios. We compare our version of the expurgated bound to both the one by Gallager and the one by Csiszár, Körner and Marton (CKM). For expurgated ensembles of fixed composition codes over finite alphabets, our basic expurgated bound coincides with the CKM expurgated bound, which is in general tighter than Gallager’s bound, but with equality for the optimum type class of codewords. Our method, however, extends beyond fixed composition codes and beyond finite alphabets, where it is natural to impose input constraints (e.g., power limitation). In such cases, the CKM expurgated bound may not apply directly, and our bound is in general tighter than Gallager’s bound. In addition, while both the CKM and the Gallager expurgated bounds are based on Bhattacharyya bound for bounding the pairwise error probabilities, our bound allows the more general Chernoff distance measure, thus giving rise to additional improvement using the Chernoff parameter as a degree of freedom to be optimized.

Index Terms: Expurgated exponents, expurgated ensembles, Bhattacharyya distance, Chernoff distance, random energy model.

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