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On the Data Processing Theorem in the Semi–Deterministic Setting

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Abstract

Data processing lower bounds on the expected distortion are derived in the finite–alphabet semi–deterministic setting, where the source produces a deterministic, individual sequence, but the channel model is probabilistic, and the decoder is subjected to various kinds of limitations, e.g., decoders implementable by finite–state machines, with or without counters, and with or without a restriction of common reconstruction with high probability. Some of our bounds are given in terms of the Lempel–Ziv complexity of the source sequence or the reproduction sequence. We also demonstrate how some analogous results can be obtained for classes of linear encoders and linear decoders in the continuous alphabet case.

Index Terms: Data processing theorem, finite–state machine, Lempel–Ziv algorithm, redundancy, delay, common reconstruction.

1 Introduction

In a series of articles from the seventies and the eighties of the twentieth century, Ziv [10],[11],[12], and Ziv and Lempel [3], [13], have created a theory of universal source coding for individual sequences using finite–state machines. In particular, the work [10] focuses on universal, fixed–rate, (almost) lossless compression of individual sequences using finite–state encoders and decoders, which was then further developed to the famous Lempel–Ziv algorithm [3], [13]. In [11], the framework of [10] was extended to lossy coding for both noiseless and noisy transmission (subsections II.A and II.B of [11], respectively), and later further extended in other directions, such as incorporation of side information in the context of almost lossless compression, where the side information data