On the Corner Points of the Capacity Region of a Two-User Gaussian Interference Channel

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Abstract

This work considers the corner points of the capacity region of a two-user Gaussian interference channel (GIC). In a two-user GIC, the rate pairs where one user transmits its data at the single-user capacity (without interference), and the other at the largest rate for which reliable communication is still possible are called corner points. This paper relies on existing outer bounds on the capacity region of a two-user GIC to derive informative bounds on these corner points for the case of two-sided weak interference (i.e., when both cross-link gains in standard form are positive and below 1). The informative bounds on the corner points are given in closed-form expressions, and a refinement of these bounds that refers to the possible increase in the rate of the second user when the first user operates at rate that is ε away from the single-user capacity is also considered. Upper and lower bounds on the gap between the sum-rate and the maximal achievable total rate at the two corner points are derived. This is followed by an asymptotic analysis analogous to the study of the generalized degrees of freedom (where the SNR and INR scalings are coupled), leading to asymptotic characterizations of this gap. The characterization is tight for the whole range of this scaling, and the simple upper and lower bounds on this gap are asymptotically tight in the sense that they achieve this asymptotic characterization. The upper and lower bounds on this gap are improved for finite SNR and INR, and these improvements are exemplified numerically.

1. Introduction

The two-user Gaussian interference channel (GIC) has been extensively treated in the literature during the last four decades (see, e.g., [8, Chapter 6] and references therein). For completeness and to set notation, the model of the two-user GIC in *standard form* is introduced shortly: this discrete-time, memoryless interference channel is characterized by the following relation between the two inputs (X_1, X_2) and corresponding two outputs (Y_1, Y_2):

$$Y_1 = X_1 + \sqrt{a_{12}}\, X_2 + Z_1 \quad (1)$$

$$Y_2 = \sqrt{a_{21}}\, X_1 + X_2 + Z_2 \quad (2)$$

where the cross-link gains a_{12} and a_{21} of the GIC are time-invariant, the inputs and outputs are real valued, and Z_1 and Z_2 denote additive Gaussian noise samples that are independent of the inputs. Let $X_1^n \triangleq (X_{1,1}, \ldots, X_{1,n})$ and $X_2^n \triangleq (X_{2,1}, \ldots, X_{2,n})$ be the two transmitted codewords across the channel where $X_{i,j}$ denotes the symbol that is transmitted by user i at time instant j (here, $i \in \{1, 2\}$ and $j \in \{1, \ldots, n\}$). No cooperation between the transmitters is allowed (so X_1^n, X_2^n are independent), nor between the receivers; however, it is assumed that the receivers have full knowledge of the codebooks used by both users. The power constraints on the inputs are given by $\frac{1}{n}\sum_{i=1}^n \mathbb{E}[X_{1,i}^2] \leq P_1$ and $\frac{1}{n}\sum_{i=1}^n \mathbb{E}[X_{2,i}^2] \leq P_2$ where $P_1, P_2 > 0$. The additive Gaussian noise samples of Z_1^n and Z_2^n are i.i.d. with zero mean and unit variance, and they are also independent of the inputs X_1^n and X_2^n. Furthermore, Z_1^n and Z_2^n can be assumed to be independent since the capacity region only depends on the marginal conditional *pdf*s $p(y_i|x_1, x_2)$ for $i = 1, 2$ (due to the non-cooperation between the receivers). Perfect synchronization between the pairs of transmitters and receivers is assumed, which implies that the capacity region is convex (since time-sharing between the users is possible).

In spite of the simplicity of this model, the exact characterization of the capacity region of a GIC is yet unknown, except for strong ([11], [18]) or very strong interference [2]. Specifically, the corner points of the capacity region have not yet been determined for GICs with weak interference; for GICs with mixed or one-sided interference, only one corner point is known (see [14, Section 6.A], [16, Theorem 2] and [19, Section 2.C]).

The operational meaning of the study of the corner points of the capacity region for a two-user GIC is to explore the situation where one transmitter sends its information at the maximal achievable rate for a single-user (in the

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