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Achieving Marton's Region for Broadcast Channels Using Polar Codes

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Abstract

This paper presents polar coding schemes for the 2-user discrete memoryless broadcast channel (DM-BC) which achieve Marton's region with both common and private messages. This is the best achievable rate region up to date, and it is tight for all classes of 2-user DM-BCs whose capacity regions are known. To accomplish this task, first we construct polar codes for the superposition and binning strategies. Then, by a combination of these two schemes, we obtain Marton's region with private messages only. Finally, we add also common information. The proposed coding schemes possess the usual advantages of polar codes, i.e., low encoding and decoding complexity and sub-exponential scaling of the error probability.

We follow the lead of the recent work by Goela, Abbe, and Gastpar, who introduce polar codes emulating the superposition and the binning schemes. In order to align the polar indices, in both the superposition and the binning scenario their solution involves some degradedness constraints that are assumed to hold between the auxiliary random variables and the channel outputs. To remove these constraints, we consider the transmission of k blocks and employ chaining constructions that guarantee the proper alignment of polarized indices. The techniques described in this work are quite general, and they can be adopted in many other multi-terminal scenarios whenever there is the need for the aligning of polar indices.

Keywords

Binning, broadcast channel, Marton's region, Marton-Gelfand-Pinsker (MGP) region, polar codes, polarization alignment, superposition coding.

I. Introduction

Polar codes, introduced by Arıkan in [1], have been demonstrated to achieve the capacity of any binary-input memoryless output-symmetric channel (BMSC) with encoding and decoding complexity $\Theta(n \log n)$, where n is the block length of the code, and a block error probability decaying like $O(2^{-n^\beta})$, for any $\beta \in (0, 1/2)$, under successive cancellation decoding [2]. A refined analysis of the block error probability of polar codes leads in [3] to rate-dependent upper and lower bounds.

The original point-to-point communication scheme has been extended, amongst others, to lossless and lossy source coding [4], [5] and to several multi-terminal scenarios, such as the Gelfand-Pinsker, Wyner-Ziv, and Slepian-Wolf problems [6], [7], the m-user multiple-access channel [8]–[12], the interference channel [13], [14], the degraded relay channel [15], [16], the wiretap channel [16]–[20], the degraded bidirectional broadcast channel with common and confidential messages [21], arbitrarily permuted parallel channels [22], and multiple description coding [23].

Goela, Abbe, and Gastpar recently introduced polar coding schemes for the m-user deterministic broadcast channel [24], [25], and for the noisy discrete memoryless broadcast channel (DM-BC) [25], [26]. For the second scenario, they considered two fundamental transmission strategies: *superposition coding*, in the version proposed by Bergmans [27], and *binning* [28]. In order to guarantee a proper alignment of the polar indices, their solution makes some degradation assumptions: in the setting of superposition coding, one of the outputs of the broadcast channel is assumed to be degraded with respect to the other; for the binning scheme, a degradedness relation is assumed to hold between some of the auxiliary random variables and the channel outputs. Note that the idea of superposition coding was first introduced by Cover [29]. Both Cover's and Bergmans's superposition schemes achieve the capacity region of

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