CCIT Report #874 December 2014

Information-Theoretic Applications of the Logarithmic Probability Comparison Bound

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December 21, 2014

Abstract

A well-known technique in estimating probabilities of rare events in general and in information theory in particular (used, e.g., in the sphere–packing bound), is that of finding a reference probability measure under which the event of interest has probability of order one and estimating the probability in question by means of the Kullback-Leibler divergence. A method has recently been proposed in [2], that can be viewed as an extension of this idea in which the probability under the reference measure may itself be decaying exponentially, and the Rényi divergence is used instead. The purpose of this paper is to demonstrate the usefulness of this approach in various information–theoretic settings. For the problem of channel coding, we provide a general methodology for obtaining matched, mismatched and robust error exponent bounds, as well as new results in a variety of particular channel models. Other applications we address include rate-distortion coding and the problem of guessing.

Index Terms: change-of-measure, error exponent, mismatch, Rényi divergence.

1 Introduction

A key approach to obtaining lower bounds on probabilities of rare events is based on the idea of a change of measure. In this approach, the underlying probability measure is replaced by a reference probability measure under which the probability of the event in question does not decay exponentially, and the exponent of the bound is given by the Kullback–Leibler (KL) divergence between the two probability measures. One then optimizes the estimate over all reference measures having the property alluded to above. This idea is standard for deriving lower bounds in large deviations theory (see, e.g., [5, p. 32]), where it is sometimes referred to as tilting. In the context of information theory it has been used in applications including (i)

*Research supported in part by the ISF (Grant 1315/12).
†Research supported in part by the ISF (Grant 412/12).