CCIT Report #875 December 2014

A differential game for a multiclass queueing model in the moderate-deviation heavy-traffic regime*

Rami Atar[†] Asaf Cohen[‡]

October 14, 2014

Abstract

We study a differential game that governs the moderate-deviation heavy-traffic asymptotics of a multiclass single-server queueing control problem with a risk-sensitive cost. We consider a cost set on a finite but sufficiently large time horizon, and show that this formulation leads to stationary feedback policies for the game. Several aspects of the game are explored, including its characterization via a (one-dimensional) free boundary problem, the semi-explicit solution of an optimal strategy, and the specification of a saddle point. We emphasize the analogy to the well-known Harrison-Taksar free boundary problem which plays a similar role in the diffusion-scale heavy-traffic literature.

AMS subject classifications: 49N70, 60F10, 60K25, 93E20

Keywords: multi-class single-server queue, moderate deviations, heavy traffic, risk-sensitive cost, the Harrison-Taksar free boundary problem

1 Introduction

This paper is concerned with the moderate-deviation (MD) scale heavy-traffic analysis of a queueing control problem. The model that is treated consists of a server that devotes its time to customers from a number of classes, where a decision maker (DM) controls the allocation of the server's effort to each of the classes. Customers of each class are kept in a buffer of finite length and so new arrivals of customers of a given class are lost when the corresponding buffer is full. The DM may reject arrivals even when buffers are not full. The cost, that is of risk-sensitive type and is rescaled at the MD regime, accounts for holding of customers in the buffers as well as for rejections. The term 'heavy traffic' refers to the imposition of a critical load condition, of the traffic intensity being close to one.

Heavy traffic analysis is traditionally carried out under the regime of *diffusion-scale deviations* (sometimes also referred to as *ordinary deviations*), where a vast variety of queueing control problems have been considered. However, there are very few results on the corresponding MD regime. This paper aims at continuing the direction started in [1] to develop an

*Research supported in part by the ISF (Grant 1315/12) and the Technion fund for promotion of research

†Department of Electrical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel

‡Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA