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Error Exponents of Typical Random Codes

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Abstract

We define the error exponent of the typical random code as the long–block limit of the negative normalized *expectation of the logarithm* of the error probability of the random code, as opposed to the traditional random coding error exponent, which is the limit of the negative normalized *logarithm of the expectation* of the error probability. For the ensemble of uniformly randomly drawn fixed composition codes, we provide exact error exponents of typical random codes for a general discrete memoryless channel (DMC) and a wide class of (stochastic) decoders, collectively referred to as the generalized likelihood decoder (GLD). This ensemble of fixed composition codes is shown to be no worse than any other ensemble of independent codewords that are drawn under a permutation–invariant distribution (e.g., i.i.d. codewords). We also present relationships between the error exponent of the typical random code and the ordinary random coding error exponent, as well as the expurgated exponent for the GLD. Finally, we demonstrate that our analysis technique is applicable also to more general communication scenarios, such as list decoding (for fixed–size lists) as well as decoding with an erasure/list option in Forney's sense.

Index Terms: error exponent, typical random code, expurgated exponent, quenched average, likelihood decoder.